1.   The curve C has equation

$$y = \frac{(x+3)(x-8)}{x}, \quad x > 0$$

(a)   Find $\dfrac{dy}{dx}$ in its simplest form.

(4)

(b)   Find an equation of the tangent to *C* at the point where *x* = 2

(4)
(Total 8 marks)

_____

2.   The curve *C* has equation

$$y = x^3 - 2x^2 - x + 9, \quad x > 0$$

The point *P* has coordinates (2, 7).

(a)   Show that *P* lies on *C*.

(1)

(b)   Find the equation of the tangent to *C* at *P*, giving your answer in the form *y* = *mx* + *c*, where *m* and *c* are constants.

(5)

The point *Q* also lies on *C*.

Given that the tangent to *C* at *Q* is perpendicular to the tangent to *C* at *P*,

(c)   show that the *x*-coordinate of *Q* is $\dfrac{1}{3}(2 + \sqrt{6})$.

(5)
(Total 11 marks)

_____

3. The curve *C* has equation

$$y = 9 - 4x - \frac{8}{x}, \qquad x > 0.$$

The point *P* on *C* has *x*-coordinate equal to 2.

(a) Show that the equation of the tangent to *C* at the point *P* is *y* = 1 − 2*x*.

(6)

(b) Find an equation of the normal to *C* at the point *P*.

(3)

The tangent at *P* meets the *x*-axis at *A* and the normal at *P* meets the *x*-axis at *B*.

(c) Find the area of triangle *APB*.

(4)
(Total 13 marks)

---

4. The curve *C* has equation $y = x^2(x - 6) + \frac{4}{x}, \; x > 0$.

The points *P* and *Q* lie on *C* and have *x*-coordinates 1 and 2 respectively.

(a) Show that the length of *PQ* is √170.

(4)

(b) Show that the tangents to *C* at *P* and *Q* are parallel.

(5)

(c) Find an equation for the normal to *C* at *P*, giving your answer in the form *ax* + *by* + *c* = 0, where *a*, *b* and *c* are integers.

(4)
(Total 13 marks)

---

5.   The curve C with equation $y = $ f(x), $x \neq 0$, passes through the point $(3, 7\frac{1}{2})$.

Given that f $'(x) = 2x + \dfrac{3}{x^2}$,

(a)   find f(x).

(5)

(b)   Verify that f$(-2) = 5$.

(1)
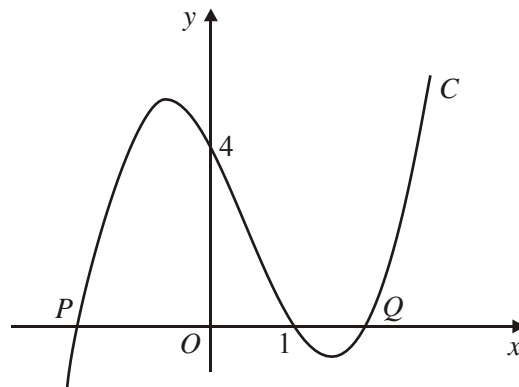
(c)   Find an equation for the tangent to C at the point $(-2, 5)$, giving your answer in
the form $ax + by + c = 0$, where a, b and c are integers.

(4)
(Total 10 marks)

_____

The curve C with equation $y = $ f(x), $x \neq 0$, passes through the point $(3, 7\frac{1}{2})$.

6.

The figure above shows part of the curve C with equation

$$y = (x - 1)(x^2 - 4).$$

The curve cuts the *x*-axis at the points P, $(1,0)$ and Q, as shown in the figure above.

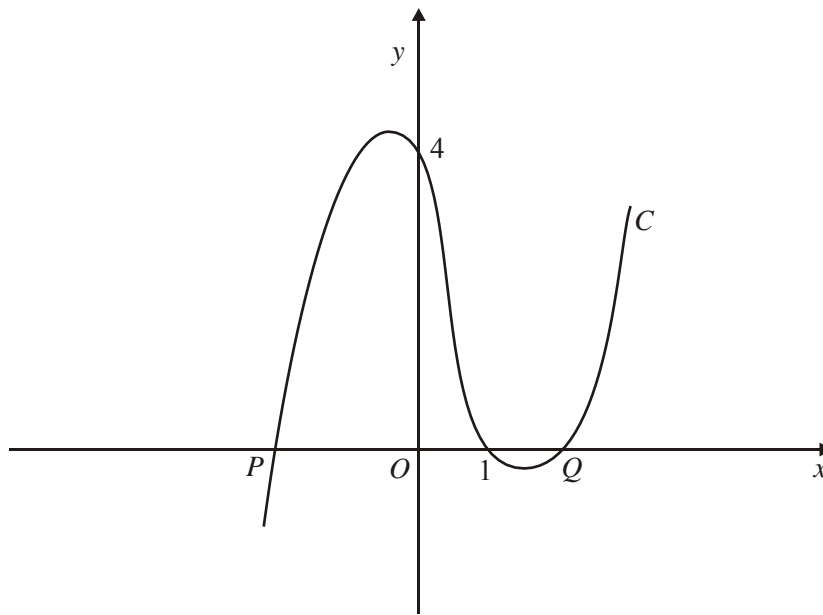(a)   Write down the *x*-coordinate of P, and the *x*-coordinate of Q.

(2)

(b)   Show that $\dfrac{dy}{dx} = 3x^2 - 2x - 4$

(3)

(c)   Show that $y = x + 7$ is an equation of the tangent to C at the point $(-1, 6)$.

(2)

The tangent to C at the point $(-1, 6)$ cuts C again at the point R.

(d)   Find the exact coordinates of R.

(5)
(Total 12 marks)

_____

7.

The figure above shows part of the curve *C* with equation

$$y = (x - 1)(x^2 - 4).$$

The curve cuts the *x*-axis at the points *P*, (1, 0) and *Q*, as shown in the figure above.

(a)   Write down the *x*-coordinate of *P*, and the *x*-coordinate of *Q*.

(2)

(b)   Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 2x - 4$.

(3)

(c)   Show that $y = x + 7$ is an equation of the tangent to *C* at the point (–1, 6).

(2)

The tangent to *C* at the point *R* is parallel to the tangent at the point (–1, 6).

(d)   Find the exact coordinates of *R*.

(5)
(Total 12 marks)

_____

8.   The curve *C* has equation $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$.

   The point *P* has coordinates (3, 0).

   (a)   Show that *P* lies on *C*.

(1)

   (b)   Find the equation of the tangent to *C* at *P*, giving your answer in the form *y = mx + c*, where *m* and *c* are constants.

(5)

   Another point *Q* also lies on *C*. The tangent to *C* at *Q* is parallel to the tangent to *C* at *P*.
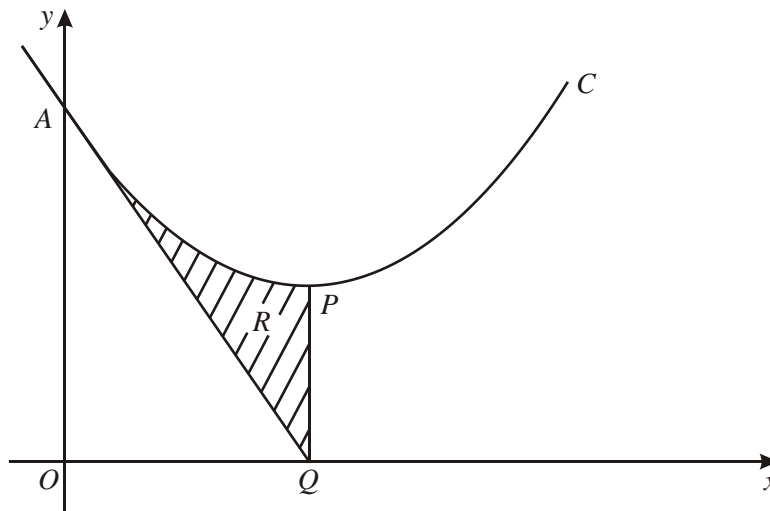
   (c)   Find the coordinates of *Q*.

(5)
(Total 11 marks)

_____

9.

The diagram above shows part of the curve C with equation $y = x^2 - 6x + 18$. The curve meets the y-axis at the point A and has a minimum at the point P.

(a)   Express $x^2 - 6x + 18$ in the form $(x - a)^2 + b$, where a and b are integers.

(3)

(b)   Find the coordinates of P.

(2)

(c)   Find an equation of the tangent to C at A.

(4)

The tangent to C at A meets the x-axis at the point Q.

(d)   Verify that PQ is parallel to the y-axis.

(1)

The shaded region R in the diagram is enclosed by C, the tangent at A and the line PQ.

(e)   Use calculus to find the area of R.

(5)
(Total 15 marks)

_____

10. The curve C has equation $y = 4x^2 + \dfrac{5-x}{x}$, $x \neq 0$. The point P on C has x-coordinate 1.

(a) Show that the value of $\dfrac{dy}{dx}$ at P is 3.

(5)

(b) Find an equation of the tangent to C at P.

(3)

This tangent meets the x-axis at the point $(k, 0)$.

(c) Find the value of k.

(2)
(Total 10 marks)

---

11. The gradient of the curve C is given by

$$\frac{dy}{dx} = (3x - 1)^2.$$

The point $P(1, 4)$ lies on C.

(a) Find an equation of the normal to C at P.

(4)

(b) Find an equation for the curve C in the form $y = f(x)$.

(5)

(c) Using $\dfrac{dy}{dx} = (3x - 1)^2$, show that there is no point on C at which the tangent is parallel to the line $y = 1 - 2x$.
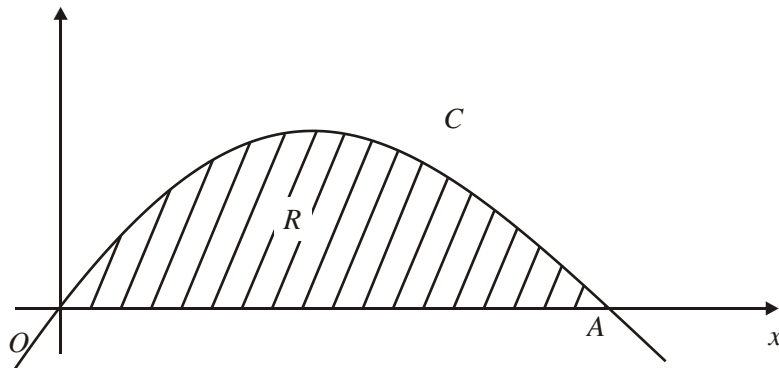
(2)
(Total 11 marks)

12.

The curve *C*, with equation $y = x(4 - x)$, intersects the *x*-axis at the origin *O* and at the point *A*, as shown in the diagram above. At the point *P* on *C* the gradient of the tangent is –2.

(a)   Find the coordinates of *P*.

(4)

The region *R*, enclosed between *C* and *OA*, is shown shaded.

(b)   Find the exact area of *R*.

(5)
(Total 9 marks)

---

13.   For the curve *C* with equation $y = x^4 - 8x^2 + 3$,

(a)   find $\dfrac{dy}{dx}$,

(2)

(b)   find the coordinates of each of the stationary points,

(5)

(c)   determine the nature of each stationary point.

(3)

The point *A*, on the curve *C*, has *x*-coordinate 1.

(d)   Find an equation for the normal to *C* at *A*, giving your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers.

(5)
(Total 15 marks)

14.
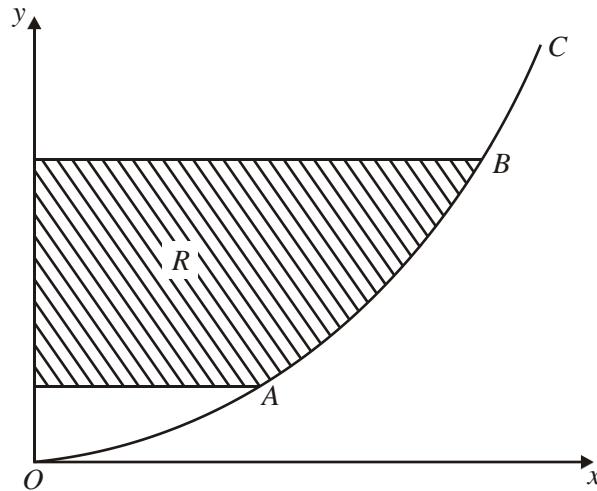
The curve *C*, shown in the diagram above, represents the graph of

$$y = \frac{x^2}{25}, \quad x \geq 0.$$

The points *A* and *B* on the curve *C* have *x*-coordinates 5 and 10 respectively.

(a)   Write down the *y*-coordinates of *A* and *B*.

(1)

(b)   Find an equation of the tangent to *C* at *A*.

(4)

The finite region *R* is enclosed by *C*, the *y*-axis and the lines through *A* and *B* parallel to the *x*-axis.

(c)   For points (*x*, *y*) on *C*, express *x* in terms of *y*.

(2)

(d)   Use integration to find the area of *R*.

(5)
(Total 12 marks)

15.
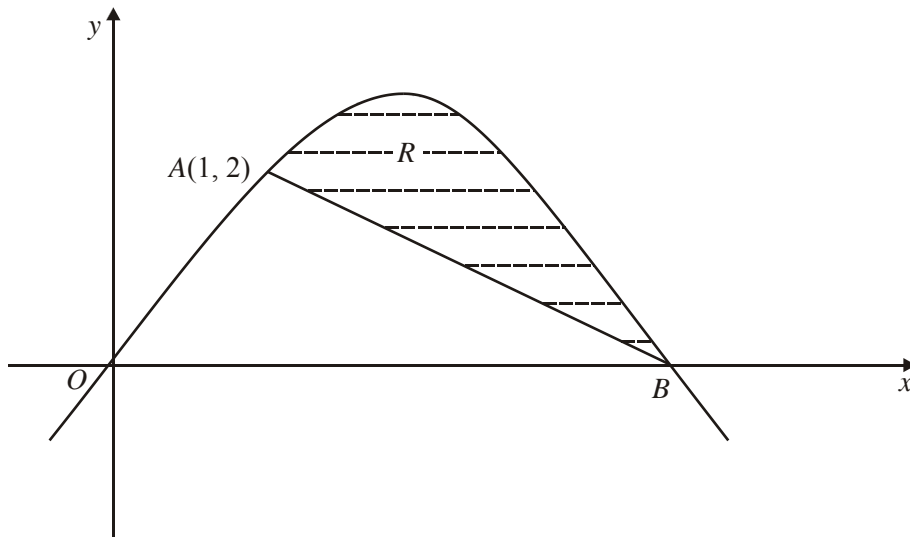
The curve *C*, shown above, has equation $y = 3x - x^2$. It passes through the origin *O* and the point *B* on the *x*-axis.

(a)   Find, in the form $ax + by + c = 0$, an equation of the normal to *C* at the point *A*(1, 2).

(5)

(b)   Show that this normal passes through the point *B*.

(2)

The shaded region *R* is bounded by the curve and the line *AB*.

(c)   Find, by integration, the area of *R*.

(7)
(Total 14 marks)

1.   (a) $y = \dfrac{x^2 - 5x - 24}{x} = x - 5 - 24x^{-1}$

(or equiv., e.g. $x + 3 - 8 - \dfrac{24}{x}$)   A1

$\dfrac{dy}{dx} = 1 + 24x^{-2}$ or $\dfrac{dy}{dx} = 1 + \dfrac{24}{x^2}$ A1   4

<u>Note</u>

1st M: Mult. out to get $x^2 + bx + c$, $b \neq 0$, $c \neq 0$ <u>and</u> dividing
by x (<u>not</u> x^2).
Obtaining one correct term, e.g. x…….. is sufficient evidence of a
division attempt.

2nd M: <u>Dependent on the 1st M</u>:

Evidence of $x^n \rightarrow kx^{n-1}$ for one x term (i.e. not just the constant
term) is sufficient). Note that mark is <u>not</u> given if, for example, the
numerator and denominator are differentiated separately.

A mistake in the 'middle term', e.g. $x + 5 - 24x^{-1}$, does not invalidate the

2nd A mark, so   A0   A1 is possible.

(b) $x = 2$: $y = -15$ Allow if seen in part (a).   B1

$\left(\dfrac{dy}{dx}\right) 1 + \dfrac{24}{4} = 7$ Follow-through from candidate's <u>non-constant</u> $\dfrac{dy}{dx}$.   B1ft

This must be simplified to a "single value".

$y + 15 = 7(x - 2)$ (or equiv., e.g. $y = 7x - 29$)
Allow $\dfrac{y + 15}{x - 2} = 7$ A1   4

<u>Note</u>

B1ft: For evaluation, using $x = 2$, of their $\dfrac{dy}{dx}$
, even if unlabelled or called y.

M: For the equation, in any form, of a straight line through

(2, '–15') with candidate's $\dfrac{dy}{dx}$ value as gradient.

Alternative is to use (2, '–15') in $y = mx + c$ to <u>find a value</u> for c, in
which case $y = 7x + c$ leading to $c = -29$ is sufficient for the A1).

(See general principles for straight line equations at the end of the scheme).

Final A: 'Unsimplified' forms are acceptable, but…

$y - (-15) = 7(x - 2)$ is A0 (unresolved 'minus minus').

[8]

2.   (a) $x = 2$: $y = 8 - 8 - 2 + 9 = 7$ (*)   B1   1

   Note

   B1   there must be a clear attempt to substitute $x = 2$ leading to 7

   e.g. $2^3 - 2 \times 2^2 - 2 + 9 = 7$

   (b) $\dfrac{dy}{dx} = 3x^2 - 4x - 1$ A1

      $x = 2$: $\dfrac{dy}{dx} = 12 - 8 - 1 (= 3)$ A1ft

      $y - 7 = 3(x - 2)$, $\underline{y = 3x + 1}$ <u>A1</u>   5

   Note

   1st for an attempt to differentiate with at least one of the given
   terms fully correct.

   1st A1   for a fully correct expression

   2nd A1ft for sub. $x = 2$ in <u>their</u> $\dfrac{dy}{dx} = (\neq y)$ accept for a correct

   expression e.g. $3 \times (2)^2 - 4 \times 2 - 1$

   2nd for use of their "3" (provided it comes from their $\dfrac{dy}{dx} = (\neq y)$

   and $x = 2$) to find equation of tangent. Alternative is to use (2, 7) in
   $y = mx + c$ to <u>find a value</u> for c. Award when $c = \ldots$ is seen.

   No attempted use of $\dfrac{dy}{dx}$ in (b) scores 0/5

   (c) $m = -\dfrac{1}{3}$ (for $-\dfrac{1}{m}$ with their m)   B1ft

      $3x^2 - 4x - 1 = -\dfrac{1}{3}, 9x^2 - 12x - 2 = 0$ or $x^2 - \dfrac{4}{3}x - \dfrac{2}{9} = 0$ (o.e.)   A1

      $\left(x = \dfrac{12 + \sqrt{144 + 72}}{18} \right) \left(\sqrt{216} = \sqrt{36}\sqrt{6} = 6\sqrt{6} \right)$ or $(3x - 2)^2$

      $6 \rightarrow 3x = 2 \pm \sqrt{6}$

      $x = \dfrac{1}{3}\left(2 + \sqrt{6} \right)$ (*)   A1cso   5

   Note

   1st for forming an equation from their $\dfrac{dy}{dx} = (\neq y)$ and their

   $-\dfrac{1}{m}$ (must be changed from m)

   1st A1   for a correct 3TQ all terms on LHS (condone missing = 0)

2nd for proceeding to $x =\ldots$ or $3x = \ldots$ by formula or completing the square for a 3TQ.
         Not factorising. Condone $\pm$

2nd A1 for proceeding to given answer with no incorrect working seen. Can still have $\pm$.

ALT   Verify (for M1A1M1A1)

1st for attempting to square need ≥ 3 correct values in $\frac{4+6+4\sqrt{6}}{9}$, 1st A1 for $\frac{10+4\sqrt{6}}{9}$

2nd Dependent on 1st in this case for substituting in all terms of their $\frac{dy}{dx}$

2nd A1cso   for cso <u>with a full comment</u> e.g. "the x co-ord of Q is ..."

[11]

3.   (a) $\left(\dfrac{dy}{dx}=\right) -4 + 8x^{-2}$ (4 or $8x^{-2}$ for   sign can be wrong)   M1A1

$x = 2 \Rightarrow$ $m = -4 + 2 = -2$

The first 4 marks <u>could</u> be earned in part (b)

$$y = 9 - 8 - \frac{8}{2} = -3$$ B1

Equation of tangent is: $y+3 = -2(x-2) \rightarrow y = 1 - 2x$ (*)   A1cso   6

<u>Note</u>

1st for 4 or $8x^{-2}$ (ignore the signs).

1st A1   for both terms correct (including signs).

2nd for substituting $x = 2$ into their $\dfrac{dy}{dx}$ (must be different from their y)

B1   for $y_P = -3$, but not if clearly found from the given equation
         of the <u>tangent</u>.

3rd for attempt to find the equation of tangent at P, follow through
         their m and y_P.

         Apply general principles for straight line equations (see end
         of scheme).
         <u>NO DIFFERENTIATION ATTEMPTED</u>: Just assuming $m = -2$
         at this stage is M0

2nd A1cso for correct work leading to printed answer (allow equivalents
with $2x$, y, and 1 terms...
                         such as $2x + y - 1 = 0$).

(b)   Gradient of normal $= \dfrac{1}{2}$ B1ft

Equation is: $\dfrac{y+3}{x-2} = \dfrac{1}{2}$ or better equivalent, e.g. $y = \dfrac{1}{2}x - 4$ M1A1   3

Note

B1ft   for correct use of the perpendicular gradient rule. Follow
        through their m, but if $m \neq -2$ there must be clear evidence
        that the m is thought to be the gradient of the tangent.
        for an attempt to find normal at P using their changed
        gradient and their y_P.
        Apply general principles for straight line equations
        (see end of scheme).

A1   for any correct form as specified above (correct
        answer only).

(c) $(A :) \dfrac{1}{2},$ $(B :) 8$ B1, B1

Area of triangle is: $\dfrac{1}{2}(x_B \pm x_A) \times y_P$ with values for

all of x_B, x_A and y_P

$$\dfrac{1}{2}\left(8 - \dfrac{1}{2}\right) \times 3 = \dfrac{45}{4} \text{ or } 11.25$$ A1   4

Note

1st B1 for $\dfrac{1}{2}$ and 2nd B1 for 8.

        for a full method for the area of triangle *ABP*. Follow through
        their x_A, x_B and their y_P, but the mark is to be awarded
        'generously', condoning sign errors..
        The final answer must be positive for A1, with negatives in
        the working condoned.

Determinant: Area $= \dfrac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \dfrac{1}{2}\begin{vmatrix} 2 & -3 & 1 \\ 0.5 & 0 & 1 \\ 8 & 0 & 1 \end{vmatrix} = \dots$ (Attempt

to multiply out required for

Alternative: $AP = \sqrt{(2-0.5)^2 + (-3)^2}$, $BP = \sqrt{(2-8)^2 + (-3)^2}$ Area

$= \dfrac{1}{2}AP \times BP = \dots$

Intersections with y-axis instead of *x*-axis: Only the M mark is
available B0 B0   A0.

[13]

4.   (a) $x = 1$: $y = -5 + 4 \underline{= -1}$, $x = 2$: $y = -16 + 2 \underline{= -14}$(can be given1stB1 for -1

in (b) or (c))2ndB1 for -14

$$PQ = \sqrt{(2-1)^2 + (-14-(-1))^2} = \sqrt{170}$$ (*)   M1A1cso   4

for attempting PQ or PQ^2 using their P and their Q.
Usual rules about quoting formulae.
We must see attempt at $1^2 + (y_P - y_Q)^2$ for
$PQ^2 = \sqrt{...}$ etc could be M1A0.

A1cso   for proceeding to the correct answer with no incorrect
working seen.

(b) $y = x^3 - 6x^2 + 4x^{-1}$

$$\frac{dy}{dx} = 3x^2 - 12x - 14x^{-2}$$ M1A1

$x = 1$: $\dfrac{dy}{dx} = 3 - 12 - 4 = -13$ M: Evaluate at one of the points

$x = 2$: $\dfrac{dy}{dx} = 12 - 24 - 1 = -13 \therefore$ ParallelA: Both correct + conclusionA1   5

1st for multiplying by x^2, the x^3 or $-6x^2$ must be correct.

2nd for some correct differentiation, at least one term must be
correct as printed.

1st A1   for a fully correct derivative.
These 3 marks can be awarded anywhere when first seen.

3rd for attempting to substitute $x = 1$ or $x = 2$ in their derivative.
Substituting in y is M0.

2nd A1 for -13 from both substitution <u>and</u> a brief comment.
The -13 must come from their derivative.

(c)   Finding gradient of normal $\left(m = \dfrac{1}{13}\right)$

$$y - -1 = \frac{1}{13}(x - 1)$$ M1A1ft
<u>$x - 13y - 14 = 0$</u>   o.e.   A1cso   4

1st for use of the perpendicular gradient rule.
Follow through their -13.

2nd for full method to find the equation of the normal
or tangent at P. If formula is quoted allow slips
in substitution, otherwise a correct substation is required.

1st A1ft   foa correct expression. Follow through their -1 and
their changed gradient

2nd A1cso   for a correct equation with = 0 and integer coefficients. This mark is dependent upon the −13 coming from their derivative in (b) hence cso.
Tangent can get M0M1A0A0, changed gradient can get M0M1A1A0 or M1M1A1A0.
Condone confusion over terminology of tangent and normal, mark gradient and equation

MR   Allow for $-\dfrac{4}{x}$ or $(x + 6)$ but not omitting $4x^{-1}$ 6r treating it as $4x$.

[13]

5.   (a) $f(x) = \dfrac{2x^2}{2} + \dfrac{3x^{-1}}{-1}(+c)$ $-\dfrac{3}{x}$ is OK   M1A1

$(3, 7\frac{1}{2})$ gives $\dfrac{15}{2} = 9 - \dfrac{3}{3} + c$ 3^2 or 3^{-1} are OK instead of 9 or $\frac{1}{3}$ M1A1f.t.

$c = -\dfrac{1}{2}$ A1   5

1st *for some attempt to integrate* $x^n \to x^{n+1}$

1st A1   *both x terms as printed or better. Ignore (+c) here.*

2nd *for use of $(3, 7\frac{1}{2})$ or (−2, 5) to form an equation for c.*
There must be some correct substitution. No +c is M0. Some changes in x terms of function needed.

2nd A1f.t.   *for a correct equation for c. Follow through their integration. They must tidy up fraction/fraction and signs (e.g. $--$ to +).*

(b) $f(-2) = 4 + \dfrac{3}{2} - \dfrac{1}{2}$ (*)   B1c.s.o   1

B1cso   *If (−2, 5) is used to find c in (a) B0 here unless they verify f(3) = 7.5.*

(c) $m = -4 + \dfrac{3}{4}, = -3.25$ A1

Equation of tangent is: $y - 5 = -3.25(x + 2)$

$4y + 13x + 6 = 0$ o.e.   A1   4

1st *for attempting $m = f'(\pm 2)$*

1st A1 *for $-\dfrac{13}{4}$ or -3.25*

2nd *for attempting equation of tangent at (−2, 5),*

$$f.t. \text{ their } m, \text{ based on } \frac{dy}{dx}.$$

2nd *A1 o.e. must have a, b and c integers and = 0.*
Treat (a) and (b) together as a batch of 6 marks.

[10]

6. (a) *x*-coordinate of *P* is –2, *x*-coordinate of *Q* is 2.   B1, B1   2

(b) $y = x^3 - x^2 - 4x + 4$ Multiplying out

$$\frac{dy}{dx} = 3x^2 - 2x - 4$$ cso   A1   3

Alternatively

Using product rule $\dfrac{dy}{dx} = 1(x^2 - 4) + (x - 1)2x$

$$= 3x^2 - 2x - 4$$ A1

(c) $x = -1 \implies m = 3 + 2 - 4 = 1$ Substituting $x = -1$ into (b)

$y - 6 = 1(x - (-1)) \implies y = x + 7$ cso   A1   2

(d) $x^3 - x^2 - 4x + 4 = x + 7$ line = curve

$x^3 - x^2 - 5x - 3 = 0$

$(x + 1)(x^2 - x - 3) = 0$ Obtaining linear × quadratic
$(x + 1)(x + 1)(x - 3) = 0$ Obtaining 3 linear factors
$R : (3, 10)$ A1, A1   5

[12]

In (d) if the correct cubic is obtained the factors can just be written
down by inspection.

Parts (c) and (d) can be done together.

On obtaining $(x + 1)^2(x - 3)$, the repeated root shows that
$y = x + 7$ is a tangent to the curve at $(-1, 6)$ and, if this is stated, the
A1 for (c) should be given at this point.

7.   (a)   –2 *(P),*   2 (*Q*)   (± 2 scores B1 B1)   B1, B1   2

   (b) $y = x^3 - x^2 - 4x + 4$ (May be seen earlier)
                                        Multiply out, giving 4 terms
          $\dfrac{dy}{dx} = 3x^2 - 2x - 4$ A1cso   3

   (c)   At $x = -1$: $\dfrac{dy}{dx} = 3(-1)^2 - 2(-1) - 4 = 1$

          Eqn. of tangent: $y - 6 = l(x - (-1))$, $y = x + 7$ A1cso   2

   (d) $3x^2 - 2x - 4 = 1$ (Equating to "gradient of tangent")

          $3x^2 - 2x - 5 = 0$ $(3x - 5)(x + 1) = 0$ $x = ...$

          $x = \dfrac{5}{3}$ or equiv.   A1

          $y = \left(\dfrac{5}{3} - 1\right)\left(\dfrac{25}{9} - 4\right)$, $= \dfrac{2}{3} \times \left(-\dfrac{11}{9}\right) = -\dfrac{22}{27}$ or equiv.   A1   5

                                                                                    [12]

   (b)   <u>Alternative</u>:
          Attempt to differentiate by product rule scores the <u>second</u>

          $\dfrac{dy}{dx} = \{(x^2 - 4) \times 1\} + \{(x - 1) \times 2x\}$

          Then multiplying out scores the <u>first</u>   with A1 if correct (cso).

   (c)   requires full method: Evaluate $\dfrac{dy}{dx}$ and use in eqn. of line through (–1,6),
          (n.b. the gradient need not be 1 for this
          <u>Alternative</u>:
          Gradient of $y = x + 7$ is 1, so solve $3x^2 - 2x - 4 = 1$, as in (d)...
          to get $x = -1$.   A1cso

   (d)   2nd and 3rd M marks are dependent on starting with $3x^2 - 2x - 4 = k$,
          where *k* is a constant.

8.   (a) $x = 3$, $y = 9 - 36 + 24 + 3$ ($9 - 36 + 27 = 0$ is OK)   B1   1

(b) $\dfrac{dy}{dx} = \dfrac{3}{3}x^2 - 2 \times 4 \times x + 8 \; (= x^2 - 8x + 8)$ A1

When $x = 3$, $\dfrac{dy}{dx} = 9 - 24 + 8 \Rightarrow m = -7$

Equation of tangent: $y - 0 = -7(x - 3)$

$\qquad\qquad\qquad\qquad y = -7x + 21$ A1 c.a.o.   5

1st *some correct differentiation ($x^n \to x^{n-1}$ for one*
term)

1st *A1 correct unsimplified (all 3 terms)*

2nd *substituting $x_p (= 3)$ in their $\dfrac{dy}{dx}$ clear evidence*

3rd *using their m to find tangent at p.*

(c) $\dfrac{dy}{dx} = m$ gives $x^2 - 8x + 8 = -7$

$\qquad\qquad (x^2 - 8x + 15 = 0)$
$\qquad\qquad (x - 5)(x - 3) = 0$
$\qquad\qquad x = (3)$ or 5 A1

$\qquad x = 5$

$\therefore y = \dfrac{1}{3}5^5 - 4 \times 5^2 + 8 \times 5 + 3$

$y = -15\dfrac{1}{3}$ or $-\dfrac{46}{3}$ A1   5

            [11]

1st *forming a correct equation " their $\dfrac{dy}{dx}$ = gradient*

of their tangent"

2nd *for solving a quadratic based on their $\dfrac{dy}{dx}$ leading*

to $x = ...$ The quadratic could be simply $\dfrac{dy}{dx} = 0$.

3rd *for using their x value (obtained from their*
quadratic) in y to obtain y coordinate. Must have one of
the other two M marks to score this.

MR
For misreading (0, 3) for (3, 0) award B0 and then M1A1
as in scheme. Then allow all M marks but no A ft. (Max 7)

9.   (a) $(x-3)^2$, +9 isw . $a=3$ and $b=9$ may just be written
              down with no method shown.   B1,   A1   3

   (b)   P is (3, 9)   B1

   (c)   A = (0, 18)   B1
         $\dfrac{dy}{dx} = 2x - 6$, at A $m = -6$ A1
         Equation of tangent is $y - 18 = -6x$ (in any form)   A1ft   4

   (d)   Showing that line meets x axis directly below P, i.e. at $x = 3$.   A1cso   1

   (e) $A = \int x^2 - 6x + 18x = [\frac{1}{3}x^3 - 3x^2 + 18x]$ A1
         Substituting $x = 3$ to find area A under curve A [=36]
         Area of R = A – area of triangle = $A - \dfrac{1}{2} \times 18 \; 3$, = 9   A1   5

         Alternative: $\int x^2 - 6x + 18 - (18 - 6x)dx$
               $= \frac{1}{3}x^3$ A1 ft
               Use $x = 3$ to give answer 9   A1

                                                                              [13]

10.   (a) $\dfrac{5-x}{x} = \dfrac{5}{x} - \dfrac{x}{x} \left(= \dfrac{5}{x} - 1\right) \left(= 5x^{-1} - 1\right)$

         $\dfrac{dy}{dx} = 8x, \; -5x^{-2}$ A1, A1

         When $x = 1$, $\dfrac{dy}{dx} = 3$ (*)   A1 cso   5

   (b)   At P, $y = 8$ B1
         Equation of tangent: $y - 8 = 3(x - 1)$ $(y = 3x + 5)$(or equiv.)   A1ft 3

   (c)   Where $y = 0$, $x = -\dfrac{5}{3}$ $(= k)$ (or exact equiv.)   A1   2

                                                                              [10]

11. (a)   Evaluate gradient at $x = 1$ to get 4, Grad. of normal $= -\dfrac{1}{m}$ $\left(= -\dfrac{1}{4} \right)$ B1,

Equation of normal: $y - 4 = -\dfrac{1}{4}(x - 1)$ $(4y = -x + 17)$ A1   4

(b) $(3x - 1)^2 = 9x^2 - 6x + 1$ (May be seen elsewhere)   B1

Integrate: $\dfrac{9x^3}{3} - \dfrac{6x^2}{2} + x \ (+C)$ A1ft

Substitute (1, 4) to find $c = \ldots, c = 3$ $(y = 3x^3 - 3x^2 + x + 3)$ A1cso   5

(c)   Gradient of given line is -2 B1
Gradient of (tangent to) C is ≥ 0 (allow > 0), so can never equal -2. B1   2

[11]

12. (a) $y = 4x - x^2$ $\dfrac{dy}{dx} = 4 - 2x$ A1

"$4 - 2x$" $= -2$, $x = \ldots$
$x = 3, y = 3$ A1   4

(b) x-coordinate of A is 4   B1

$\displaystyle\int \left(4x - x^2\right) dx = \left[\dfrac{4x^2}{2} - \dfrac{x^3}{3} \right]$ A1ft

$\left[\dfrac{4x^2}{2} - \dfrac{x^3}{3} \right]_0^4 = 32 - \dfrac{64}{3} = \dfrac{32}{3}$ $\left(= 10\dfrac{2}{3} \right)$ (or exact equivalent)   A1   5

[9]

13. (a) $\dfrac{dy}{dx} = 4x^3 - 16x$ A1   2

(b) $4x^3 - 16x = 0$
$4x(x^2 - 4) = 0$ $x = 0, \ 2, \ -2$ A2 (1, 0)
$y = 3, \ -13, \ -13$ A1   5

(c) $\dfrac{d^2y}{dx^2} = 12x^2 - 16$

$x = 0$ Max. $\left.\begin{array}{l} \\ \\ \\ \end{array}\right\}$

$x = 2$ Min.   One of these   A1 ft
$x = -2$ Min.   All three   A1   3

(d) $x = 1$: $y = 1 - 8 + 3 = -4$ B1

   At $x = 1$, $\dfrac{dy}{dx} = 4 - 16 = -12\ (m)$ B1 ft

   Gradient of normal $= -\dfrac{1}{m}$ $\left(= \dfrac{1}{12}\right)$

   $y - (-4) = \dfrac{1}{12}(x - 1)$ $x - 12y - 49 = 0$ A1   5

                                                                                  [15]

14.   (a) A: $y = 1$ B: $y = 4$ B1

   (b) $\dfrac{d\,y}{d\,x} = \dfrac{2x}{25}$ $= \dfrac{2}{5}$ where $x = 5$ A1

   Tangent: $y - 1 = \dfrac{2}{5}(x - 5)$ $(5y = 2x - 5)$ A1

   (c) $x = 5y^{\frac{1}{2}}$ B1 B1

   (d)   Integrate: $\dfrac{5y^{\frac{3}{2}}}{\frac{3}{2}}$ $\left(= \dfrac{10y^{\frac{3}{2}}}{3}\right)$ A1 ft

   $[\ ]^4 - [\ ]1 = \left(\dfrac{10 \times 4^{\frac{3}{2}}}{3}\right) - \left(\dfrac{10 \times 1^{\frac{3}{2}}}{3}\right),\ = \dfrac{70}{3}\ (23\tfrac{1}{3},\ 23.3)$ A1, A1

   <u>Alternative for (d):</u>   Integrate: $\dfrac{x^3}{75}$ A1

   Area $= (10 \times 4) - (5 \times 1) - \left(\dfrac{1000}{75} - \dfrac{125}{75}\right),\ = \dfrac{70}{3}\ (23\tfrac{1}{3},\ 23.3)$ A1, A1

   In both (d) schemes, final M is scored using <u>candidate's</u> "4" and "1".

                                                                                  [12]

15. (a) $y = 3x - x^2$

$\dfrac{dy}{dx} = 3 - 2x$

At $(1, 2)$ $\dfrac{dy}{dx} = 1$ so gradient of normal $= -1$ A1 A1 ft

Equation of normal $y - 2 = -1(x - 1)$

i.e. $x + y - 3 = 0$ A1 ft   5

(b)   From curve equation, B is $(3, 0)$ B1

and normal $x + y - 3 = 0$ is satisfied by $(3, 0)$ and $\therefore$ passes through BB1   2

(c)   Area of R = Area under curve – Area of triangle

Area of triangle $= \dfrac{1}{2} \times 2 \times 2 = 2$ units^2 B1

Area under curve $= \int (3x - x^2)\,dx = \dfrac{3x^2}{2} - \dfrac{x^3}{3}$ A1

$\left[\dfrac{3x^2}{2} - \dfrac{x^3}{3} \right]_1^3 = 13\dfrac{1}{2} - 9 - \left(\dfrac{3}{2} - \dfrac{1}{3} \right) = \dfrac{10}{3}$ A1

Area of $R = \left(\dfrac{10}{3} - 2 \right)$ units$^2 = \dfrac{4}{3}$ units^2 A1   7

[14]

1. This was a successful question for many candidates, although for some the required division by x in part (a) proved too difficult. Sometimes the numerator was multiplied by x, or x^{-1} was added to the numerator. Occasionally the numerator and denominator were differentiated separately.

    In part (b), most candidates substituted $x = 2$ into their $\frac{dy}{dx}$, but in finding the equation of the tangent numerical mistakes were common and there was sometimes confusion between the value of $\frac{dy}{dx}$ and the value of y.

2. A number of partial attempts at this question may suggest that some were short of time although the final part was quite challenging.

    Most secured the mark in part (a) although careless evaluation of $2 \times (2)^2$ as 6 spoiled it for some. Apart from the few who did not realise the need to differentiate to find the gradient of the curve, and hence the tangent, part (b) was answered well. Some candidates though thought that the coefficient of x^2 (the leading term) in their derivative gave them the gradient. There was the usual confusion here between tangents and normals with some candidates thinking that $\frac{dy}{dx}$ gave the gradient of the normal not the tangent. In part (c) many knew they needed to use the perpendicular gradient rule but many were not sure what to do. A common error was to find the equation of a straight line (often the normal at P) and then attempt to find the intersection with the curve. Those who did embark on a correct approach usually solved their quadratic equation successfully using the formula, completing the square often led to difficulties with the x^2 term, but a few provided a correct verification.

3. Responses to this question varied considerably, ranging from completely correct, clear and concise to completely blank. Most candidates who realised the need to differentiate in part (a) were able to make good progress, although there were occasionally slips such as sign errors in the differentiation. A few lost marks by using the given equation of the tangent to find the y-coordinate of P. Those who used no differentiation at all were limited to only one mark out of six in part (a). Even candidates who were unsuccessful in establishing the equation of the tangent were sometimes able to score full marks for the normal in part (b).
    Finding the area of triangle APB in part (c) proved rather more challenging. Some candidates had difficulty in identifying which triangle was required, with diagrams suggesting intersections with the y-axis instead of the x-axis. The area calculation was sometimes made more difficult by using the right angle between the tangent and the normal, i.e. $\frac{1}{2}(AP \times BP)$, rather than using AB as a base.

4.   A number of candidates did not attempt this question or only tackled part of it. In part (a) the *y*-coordinates were usually correct and the distance formula was often quoted and usually used correctly to obtain *PQ,* although some drew a diagram and used Pythagoras' theorem. Most of the attempts at part (b) gained the first two method marks but the negative index was not handled well and errors in the derivative were sometimes seen. Those with a correct derivative were usually able to establish the result in part (b). Some candidates thought that they needed to find the equations of the tangents at *P* and *Q* in order to show that the tangents were parallel and this wasted valuable time. It was unfortunate that the gradient of *PQ* was also equal to – 13 and a number of candidates did not attempt to use any calculus but simply used this gradient to find the tangents and tackle part (c). Errors in the arithmetic sometimes led to the abandonment of the question at this point which was a pity as some marks could have been earned in part (c).
        Those who did attempt part (c) were usually aware of the perpendicular gradient rule and used it correctly to find the equation of the normal at *P*. Mistakes in rearranging the equation sometimes led to the loss of the final mark but there were a good number of fully correct solutions to this part and indeed the question as a whole.

5.   This question was not always answered well. Most candidates knew that integration was required in part (a) and usually they scored both the marks but many forgot to include a constant of integration. Many still did not realize their error even when they obtained f(–2)=5.5 and this proved to be a costly mistake. The candidates could still complete part (c) even if they had made mistakes in part (a) and most attempted this part. The most common error was to find the equation of the chord between the points (–2, 5) and (3, 7.5). Those who realized that f'(–2) was required often had trouble with the arithmetic and some thought that the gradient of the normal was required. A few candidates used (3, 7.5) instead of (–2, 5) in part (c). Those candidates who successfully negotiated these pitfalls usually gave their answer in the required form but there were few fully correct solutions to this part.

6.   The first three parts of the question were well done although a few did not realise that the result of part (b) was needed to produce a satisfactory solution to part (c). Part (d) proved very difficult for the majority. Most could equate the curve to the line and reduce the problem to solving the cubic $x^3 - x^2 - 5x - 3 = 0$ but many left it there and others tried laborious methods of searching for roots using the factor theorem not recognising that the conditions of the question imply that $x = -1$ must be a root of the equation. If it is recognised that $x = -1$ must be a repeated root then $x^3 - x^2 - 5x - 3 = (x + 1)^2 (x - 3)$ can just be written down but only 4 or 5 candidates saw this.

7.   In part (a), a surprising number of candidates were unable to find both values. Although $x = 2$ for Q was usually seen, $x = -2$ for P was sometimes not realised.

Part (b) required the expansion of $(x - 1)(x^2 - 4)$ and then the verification of its derivative, and this was well done by the vast majority. In part (c), however, where candidates had to show that $y = x + 7$ is an equation of the tangent to the curve at the point $(-1, 6)$, many simply substituted the x and y values into the equation of the curve and/or the given equation of the tangent. Use of the derivative was needed here.

In part (d), weaker candidates often had no idea how to find the other point on the curve with a parallel tangent. A common wrong approach was to use the method for finding the point at which the tangent intersects the curve again. Those who did start correctly often proceeded well, but accuracy in the calculation of the y-coordinate was frequently lacking. Sometimes the value of x was substituted into the equation of the tangent, or into the derivative, rather than into the equation of the curve.

8.   A number of candidates did not answer this question which could have been due to a lack of time or a reflection of the difficulty of this question. Part (a) was usually answered correctly but some evaluated $\frac{1}{3}3^3$ to get 3. Most differentiated in part (b) to find the gradient although a minority tried simply substituting values into $y = mx + c$. A number of candidates were unable to calculate the gradient correctly at $x = 3$ ($9 - 24 + 8$ was often incorrectly evaluated). Many had no idea how to tackle part (c) but those that did scored well although a surprising number followed a correct substitution of $x = 5$ with $y = 41\frac{2}{3} - 57$ but then gave the final answer as $y = 15\frac{1}{3}$ instead of $y = -15\frac{1}{3}$.

9.   (a)   This was generally answered very well. Many candidates scored full marks, with others gaining B1,   A0, for answers such as $(x-3)^2-9$, or -27, or +27

(b)   The majority of candidates did not use their (a) to get the answer in (b) but used differentiation. This meant that most candidates had this part correct, even if they had (a) incorrect or didn't attempt it.

(c)   Again this part was well answered, especially by those who had done the differentiation in (b) as they went on to get the gradient of -6, and used (0,18) to get the equation of the line. Unfortunately, some assumed the coordinates of Q at (3,0), and found the gradient using the points A and Q, so didn't gain many marks at all in this part.

(d)   Generally if candidates had answered part (c) correctly, they were able to do part (d) as well, although quite a few lost credibility because they stated that the gradient was 0. Many compared the x coordinates and deduced the line was parallel to the y axis and gained the credit.

(e)   This was very well answered. Many candidates had full marks in this part even if they hadn't scored full marks earlier. A few candidates made the mistake of using the y value of 9 instead of the x value of 3 in the integral. Other errors included using a trapezium instead of a triangle, and some candidates made small slips such as the 18 being copied down as an 8 or integrating the 6x to get 6x/2, or $6x^2$. It is possible that these candidates were short of time.

10. The main difficulty in part (a) of this question was the inability to divide correctly to express $\frac{5-x}{x}$ as two separate terms. Although $5x^{-1}$ was commonly seen, $\frac{x}{x}$ frequently became zero. This mistake still led to a value of 3 for the derivative, so usually went unnoticed by candidates (but not by examiners). The given answer in (a) enabled most candidates to proceed with parts (b) and (c), where there were many good solutions. Some thought, however, that for $\frac{dy}{dx} = 3$, the gradient of the tangent was $-\frac{1}{3}$, showing confusion between tangents and normals. Occasional slips were seen in part (c), but most candidates realised that the use of $y = 0$ was required to find the value of k.

11. In this question the demands parts (a) and (b) were sometimes confused, with candidates not making it clear whether they were finding the equation of the curve or the normal. The tangent equation was also frequently seen as a solution to part (a). Apart from these problems, many good solutions were produced for part (a), and most candidates were able to expand $(3x-1)^2$ correctly and to proceed to integrate in part (b) to find the equation of the curve. Occasionally candidates failed to use the fact that $P(1, 4)$ was on the curve and left their answer as $y = 3x^3 - 3x^2 + x + C$, losing two marks. Part (c) proved difficult for many candidates. Some omitted it, others stated the gradient of the given straight line and proceeded no further, and some only considered the gradient of the curve at a specific point. There were, however, some very good solutions to this part, in which candidates explained clearly why there was no point on the curve at which the tangent was parallel to the line. A popular approach was to form a quadratic equation by equating $(3x-1)^2$ to -2, then to show that the equation had no real solutions. Mistakes included equating $(3x-1)^2$ either to $-2x$ or to $1-2x$.

12. In part (a) of this question, some candidates failed to appreciate the need to differentiate and therefore made little progress. Such candidates often tried, in various ways, to use an equation of a straight line with gradient -2, perhaps inappropriately passing through the point A. Those who did differentiate were often successful, although some ignored the -2 and equated their derivative to zero.

There were many completely correct solutions to part (b). Integration techniques were usually sound, but a few candidates had difficulty in finding the correct limits, especially where they looked for a link between parts (a) and (b).

13. The differentiation in part (a) was completed successfully by nearly all candidates, but some made no further progress, perhaps not understanding the meaning of "stationary point". Most, however, knew in part (b) that they needed to use $\frac{dy}{dx} = 0$, but a very common mistake in solving $4x^3 - 16x = 0$ was to omit one or two of the three possible solutions. Some candidates also failed to find the corresponding y coordinates. Use of the second derivative in part (c) was the preferred method to determine the nature of the stationary points, and some good solutions were seen here, although some candidates tried equating their second derivative to zero either in this part or part (b). Finding the equation of the normal in part (d) proved difficult for some candidates. While most found the y-coordinate of C, some did not realise that they needed to use $\frac{dy}{dx}$ to find the gradient, then others found the equation of the tangent instead of the normal. Where methods *were* correct, algebraic or arithmetic slips often led to the loss of at least one mark, but the correct answer in the required form was not uncommon.

14. Although many candidates found this question difficult, those who were competent in algebra and calculus often produced excellent, concise solutions.

Part (a) caused very little difficulty, almost everyone scoring the one available mark. In part (b), however, attempts at finding the equation of the tangent were disappointing. Some candidates made no attempt to differentiate, while others differentiated correctly to get $\frac{2x}{25}$, but failed to find the gradient at $x = 5$, using instead $\frac{2}{25}$, or even $\frac{2x}{25}$, as the gradient m in the equation of the tangent.

Although some had no idea what to do in part (c), the majority were able to express x as $\sqrt{25y}$ or equivalent. Here, it was not always clear whether the square root extended to the y as well as the 25, but a correct expression in part (d) gained the marks retrospectively.

The expression of x in terms of y was, of course, intended as a hint for part (d), but many candidates happily ignored this and integrated y with respect to x rather than x with respect to y. Using this approach, just a few were able to find the area of the required region by a process of subtracting the appropriate areas from a rectangle of area 40, but it was very rare to see a complete method here. Some candidates wrongly used 5 and 10 as y limits (or 1 and 4 as x limits). Those integrating 5Öy with respect to y were very often successful in reaching the correct answer.

15. No Report available for this question.